III. " Rectification of Logarithmic Errors in the Measurements of Two Sections of the Meridional Arc of India." In a Letter to Professor STOKES, Sec. R.S. By Colonel EVE-REST, F.R.S. Received December 22, 1858.

It will be in your recollection, that some years ago, at the request of the Court of Directors of the East India Company, I compiled from the General Report of the Great Trigonometrical Survey of India, a work entitled " An Account of the Measurement of two Sections of the Meridional Arc of India," executed by myself and my assistants, an impression of which (printed in 1847) was presented to the Royal Society.

In reference to the work in question, I now have to mention that in the computation of the meridional triangles (*vide* pp. 240 to 248) there have been some errors committed in taking out the logarithms of the twelfth triangle (p. 241) and the twentieth triangle (p. 243); and as from the nature of that series, the purpose of which is to project the sides of the principal triangles on the meridian, any such error must run through all the computations subsequent to it, I have had the whole series recomputed, and now forward for submission to the Royal Society, a sheet containing a revised synopsis (*vide* p. 248), such as it would have been, but for the errors adverted to, to which it is my wish that the utmost publicity should be given.

I must here say, that I owe the detection and correction of the said errors entirely to the industry of Colonel Waugh, F.R.S., and the able computing establishment at his disposal; for myself it remains only to urge, that though mistakes of this nature are not cre-ditable, but, on the contrary, much to be regretted, yet it is all but impossible for the chief of two departments, that of Surveyor-General of India, and that of Superintendent of the Great Trigonometrical Survey of India, each involving a vast array of business peculiar to itself, to enter into all the minutiæ of computation; I took the precaution to have every portion of the work gone over by two computers acting independently, and it is singular that both should have fallen into the same errors.

As the existence of these errors, in the instance of the northern of the two sections or series A, would naturally lead to the supposition that like errors might lurk in series B (pp. 46 to 53), an equally

rigorous recomputation has been kindly instituted by Colonel Waugh in the latter case ; and as no mistake is found therein, the synopsis B (p. 53) will need no alteration on that account.

I have now to advert to another subject, also relating to the work in question. In trigonometrical operations, I need hardly mention that the absolute lengths of all linear quantities depend on those of the measured bases reduced to their equivalents at the level of the sea, the reduction on which account may be expressed by $\dfrac{B h}{R}$, where B is the measured base, h its mean height above the sea, and R the radius of curvature : now as we do not know *a priori* the value of h, unless the measurements be actually made on the sea-shore, the only mode of commencing the work of computation is to assume the nearest value we are in possession of, and when the operations are connected with the sea-coasts, to apply a correction for any excess or defect of our assumption.

Damargida, the southernmost point included in my book of 1847, is an inland station, the height of which is given in the general report of my predecessor, Colonel Lambton, at 2026 feet ; and as all the triangulation to the south of that station had been concluded in 1815, nearly four years before I joined the department, I had no choice but to refer to that as one of my established data : my computations start from the Sironj base, and on reaching Damargida it appeared that the assumed value of h, which I had used in correcting that base line, was 24·5 feet in excess of what it ought to have been, for which the correction is applied of 2·578 feet to the linear value of the terrestrial arc in synopsis B, and 2·295 in synopsis A.

Colonel Lambton's operations, from which the height of Damargida station is determined, abut on the sea-coast at three different points, viz. Madras on the east coast, Mangalore on the west coast, and Cape Comorin on the southernmost extremity of the Peninsula, and the results thence derived were at that time the most trustworthy data I had access to.

Subsequent to the completion of my computations, the western longitudinal series which connects Damargida with the sea-coast near Bombay, was finished by Captain Jacob, then one of my assistants, and there is a note on that subject at page clxxi. of my book of 1847, which is, I presume, quite sufficient to prepare any

geodist for the probability of a future correction being needed : as to Captain Jacob's performance, I may distinctly state that I place the greatest confidence in its accuracy, for he was furnished with an excellent altitude and azimuth instrument by Dollond, which had a good vertical circle, and he was not only a highly-talented mathematician, but a most careful and skilful observer.

Since I left India, in December 1843, the trigonometrical operations have abutted on the sea-coast at two other places, viz. at Calcutta, by the completion of my north-east longitudinal series, and at Karachi, by Colonel Waugh's western longitudinal series; so that now there are several additional data relative to the numerical value of h, which it becomes necessary to take into consideration in the determination of the proper corrections to be applied to the measured bases, and the lengths of my two terrestrial arcs, as given in synopses B and A.

I have for some time been in correspondence with Colonel Waugh on this subject, and in due time hope to be able to communicate what final conclusion we arrived at; but, in the meanwhile, as it may be interesting to the Fellows of the Royal Society to know the provisional state of the question, I here subjoin some extracts of his last received communication.

" By the completion of the Calcutta meridional series, as well as the north-east longitudinal series, we obtain a continued chain of triangulation, extending from the sea-level near Calcutta along the Calcutta meridian to the Sonakoda base, thence along the north-east longitudinal series to Dehra Dun, thence down the great arc, and along the Bombay longitudinal series to the sea-level at Bombay. The result of trigonometrical levelling along this course of 2127 miles, comprising 1171 miles of hills, and 956 miles of plains, becomes verified ; the discrepancy in height being 6·76 feet or 0·003 foot per mile.

" By the completion of the great longitudinal series, we get another continuous chain of triangulation from the sea-level near Calcutta to the same level at Karachi. This chain embraces the Calcutta meridional series, the north-east longitudinal series, the northern section of the great arc from Dehra Dun to Sironj, and the great longitudinal series from Sironj to Karachi. This series is 2082 miles in length, comprising 1041 miles of plains, and 1041 miles of

hills, and the error by trigonometrical levelling is 16·50 feet, or 0·008 foot per mile.

" When the great Indus series is finished, the sea-level will be brought up to the Chueh base line, and from thence along the north-west Himalayan series to Dehra Dun. The result of this series will give another verification, which will be especially valuable, as it is intended to level along the flat valley of the Indus by special level-ling operations. It is also proposed to carry special levelling ope-rations from Dehra Dun to Calcutta. Pending the completion of these undertakings, it has been found necessary to correct provi-sionally the results given by trigonometrical levelling, in order to disperse the discrepancies above shown, and reduce the results of the three data for sea-level near Calcutta, Bombay, and Karachi. The results, so reduced, become comparative, *inter se*, and are required for record on the general maps."

The upshot of all this is, that Colonel Waugh has provisionally applied the following corrections to the two sections of the great arc, as given in synopsis A, p. 248, and synopsis B, p. 53, of my book of 1847, in lieu of those formerly applied; and I must say that I think they will be nearer the truth than those given in that book, because the discrepancy specified in the note at the foot of p. clxxi., taken into combination with all the results of the general trigono-metrical survey since obtained, certainly point to some error in Colonel Lambton's operations to the south of Damargida. The two sections, applying Colonel Waugh's corrections, will stand as follows :—

SECTION A.

	Feet.
Length of the arc deduced from the Sironj base	1961155·422
Add correction for 111·14 feet in height at ditto	10·412
Add correction η, as explained in p. clxxi.	12·387
Total arc, Kalianpur to Kaliana, by provisional data	1961178·221

SECTION B.

	Feet.
Length of the arc deduced from the Sironj base	2202914·322
Add correction for 111·14 feet in height at ditto	11·695
Add correction η, as explained in p. clxxi.	8·679
Total arc, Kalianpur to Damargida, by provisional data	2202934·696

Series A, Synopsis, showing the length of the Terrestrial Arc comprised between Kalianpur and Kaliana.

No.	Points of Intersection.	Distances in Feet.		Results.
		Positive.	Negative.	
1	Kalianpur to μ^i	78993·100	....	}304969·223
2	μ^i to μ^{ii}	....	53204·174	
3	μ^{ii} ,, μ^{iii}	164528·038	....	
4	μ^{iii} ,, μ^{iv}	....	4238·479	
5	μ^{iv} ,, μ^{v}	118890·738	....	
6	μ^{v} ,, μ^{vi}	....	17782·494	}297104·734
7	μ^{vi} ,, μ^{vii}	90318·962	....	
8	μ^{vii} ,, μ^{viii}	22495·673	....	
9	μ^{viii} ,, μ^{ix}	128257·147	....	
10	μ^{ix} ,, μ^{x}	....	21907·376	
11	μ^{x} ,, μ^{xi}	102857·688	....	
12	μ^{xi} ,, p^i	....	7134·866	
13	p^i ,, μ^{xii}	11359·821	....	}427109·613
14	μ^{xii} ,, μ^{xiii}	66041·591	....	
15	μ^{xiii} ,, μ^{xiv}	41613·757	....	
16	μ^{xiv} ,, μ^{xv}	47331·477	....	
17	μ^{xv} ,, μ^{xvi}	62497·787	....	
18	μ^{xvi} ,, μ^{xvii}	49466·835	....	
19	μ^{xvii} ,, μ^{xviii}	135674·688	....	
20	μ^{xviii} ,, p^{ii}	13123·657	....	
21	p^{ii} ,, μ^{xix}	....	11798·757	}328204·457
22	μ^{xix} ,, p^{iii}	113184·729	....	
23	p^{iii} ,, μ^{xx}	....	67673·046	
24	μ^{xx} ,, μ^{xxi}	166111·919	....	
25	μ^{xxi} ,, μ^{xxii}	....	47224·277	
26	μ^{xxii} ,, μ^{xxiii}	175603·889	....	
27	μ^{xxiii} ,, μ^{xxiv}	....	4171·424	}320212·458
28	μ^{xxiv} ,, μ^{xxv}	136368·819	....	
29	μ^{xxv} ,, μ^{xxvi}	....	43939·700	
30	μ^{xxvi} ,, μ^{xxvii}	169944·819	....	
31	μ^{xxvii} ,, μ^{xxviii}	....	45108·400	
32	μ^{xxviii} ,, μ^{xxix}	107118·344	....	
33	μ^{xxix} ,, μ^{xxx}	....	2623·485	
34	μ^{xxx} ,, μ^{xxxi}	94298·944	....	}283554·937
35	μ^{xxxi} ,, μ^{xxxii}	42013·658	....	
36	μ^{xxxii} ,, μ^{xxxiii}	79728·356	....	
37	μ^{xxxiii} ,, μ^{xxxiv}	66898·846	....	
38	μ^{xxxiv} ,, p^{iv}	3238·631	....	
39	p^{iv} ,, p^{iv}	....	0·013	

Length of the Arc, deduced from the Seroz Base...... 1961155·422
Add correction for 24·5 feet difference of height...... 2·295
Add correction η, as explained elsewhere............... 13·108

Total Terrestrial Arc from the two Bases in feet...... 1961170·825

I must call attention to the fact, that all the remarks of Colonel Waugh have reference to the operations carried on since 1832 by me, himself, and our assistants, and are quite disconnected from Colonel Lambton's results to the south of Damargida, which were deduced in the early part of 1815. Now, as the two sets of operations unite at Damargida, it would be illogical to correct the southern portion of the Indian arc by one value of h, and the northern portion by another value of h; for, manifestly, no spot on the earth's surface can have two distinct heights above the sea differing from each other by 100 feet or so at the same instant.

It is rather a delicate matter for me to speak in terms of comment of the labours of my predecessor, and I had much rather that anybody else should undertake the invidious task; but thus much is very certain, that the instruments employed prior to 1832, were not such as we should use with confidence now-a-days, whilst those in use since that period are certainly not surpassed by any in existence.

In 1842 I was sensibly alive to the probable inaccuracy of the operations between Damargida and Cape Comorin, and, as the only effectual remedy, proposed to the Government of India to revise that work with the new instruments; but the proposal was rejected; so that instead of forming, as it would and ought to have done, one unbroken series from Cape Comorin to the Himalayan Mountains, the great arc of India now consists of two distinct patches; one to the south of Damargida for ever uncertain as to its unit of measure, and executed with instruments which we should now pronounce to be crazy and unserviceable; whilst the other to the north depends on a unit tolerably well defined, and was performed with instruments as perfect as can be desired. Of course, it is not for me to offer any remarks about the propriety of the decision of the Right Honourable the Governor-General of India in Council, and the Court of Directors of the East India Company, of those days, in a communication of this nature, except that, as far as science is concerned, perhaps it will now and hereafter be lamented that all my arguments for a revision were urged in vain.

It will appear, from what I have above stated, that the question of the amount of correction to be applied to my two sections A and B, is not yet finally settled, and is still uncertain to the extent of

about 10 feet; for which reason I have employed in the synopsis A, now forwarded, the same values as before, correcting only the obvious errors to which I have drawn attention in the second paragraph of this letter; at the same time I have endeavoured to put within the reach of those who interest themselves in the problem of the figure of our planet, all the data at my command, and if any fresh light should hereafter be thrown on the subject, shall be very happy to communicate it.

<div align="right">GEORGE EVEREST.</div>

IV. "On the Thermodynamic Theory of Steam-engines with dry saturated Steam, and its application to practice." By W. J.MACQUORN RANKINE, C.E., LL.D., F.R.S.S.L. & E., Pres. Inst. Eng. Scot., Regius Professor of Civil Engineering and Mechanics in the University and College of Glasgow. Received December 27, 1858.

(Abstract.)

In 1849 it was demonstrated, contemporaneously and independently, by Professor Clausius and the author of this paper, from the laws of thermodynamics, that when steam or other saturated vapour in expanding performs work, and receives no heat from without, a portion of it must be liquefied.

That theoretical conclusion has since been confirmed by practical experience.

The principal effect of the "steam-jacket" invented by Watt is to prevent that liquefaction.

The presence of liquid water in any considerable quantity in the cylinder of a steam-engine acts injuriously, by taking heat from the steam while it is being admitted, and giving out that heat to the steam which is about to be discharged. Most of the heat so transferred is wasted.

The only *exact thermodynamic* formulæ for the work of steam hitherto published (by the author in the Phil. Trans. 1854, and by Professor Clausius in Poggendorff's 'Annalen,' 1856), are adapted to steam which receives no heat in expanding.

The present paper, after recapitulating the general equation of thermodynamics, and the special formulæ for the pressure, volume,